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Neutrino mass matrix with approximate flavor symmetry

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Abstract: Phenomenological implications of neutrino oscillations implied by recent experimental data on pattern of neutrino mass matrix are disscussed. It is shown that it is possible to have a neutrino mass matrix which shows approximate flavor symmetry; the neutrino mass differences arise from flavor violation in off-diagonal Yukawa couplings. Two modest extensions of the standard model, which can embed the resulting neutrino mass matrix have also been discussed.

Keywords: Beyond Standard Model, Neutrino Physics.

The data [1]–[5] from solar and atmospheric neutrino experiments provide evidence for neutrino mass and mixing. As such lepton flavor is violated providing evidence for physics beyond the standard model (SM). The present experimental data can be explained by neutrino oscillations with mass squared differences and mixing angles having following best fit values [6]

$$\begin{aligned} \Delta m^2_{\text{atm}} &\equiv \Delta m^2_{23} = (2.7 \pm 0.4) \times 10^{-3}\,\text{eV}^2 \\ \sin^2\theta_{23} &\equiv \sin^2 2\theta_1 = 1.00 \pm 0.04 \end{aligned} \tag{1}$$

$$\begin{aligned} \Delta m^2_{\text{solar}} &\equiv \Delta m^2_{12} = (7.1 \pm 0.6) \times 10^{-5}\,\text{eV}^2 \\ \tan^2\theta_{12} &\equiv \tan^2\theta_3 = 0.45 \pm 0.06\,. \end{aligned} \tag{2}$$

Further the CHOOZ [7] experiment gives a bound

$$|U_{e3}|^2 \equiv \sin^2\theta_2 < 4 \times 10^{-2}\,. \tag{3}$$

The purpose of this paper is to interpret these results in terms of small off-diagonal perturbations of a degenerate diagonal mass matrix in flavor basis for light Majorana neutrinos. In this approach there is no fundamental distinction between masses of neutrinos of different flavor; the mass differences arise from small flavor violation of off diagonal Yukawa coupling constants. Further it is shown that the neutrino mass differences do not in anyway constraint the absolute value of neutrino mass. The constraint on it will come from neutrinoless double-beta decay experiments, cosmology and direct laboratory experiments, e.g. tritium beta-decay.

Lastly we will try to embed the resulting neutrino mass matrix in two different modest extensions of the standard model. In one version the minimal standard model of particle interactions is extended to include a Higgs triplet to introduce Majorana mass. In the other the standard electroweak gauge group is extended to $\mathrm{SU}_L(2) \times \mathrm{U}_e(1) \times \mathrm{U}_\mu(1) \times \mathrm{U}_\tau(1)$ leading to off-diagonal mass matrix for light (Majorana) neutrinos which can act as a small perturbation to a degenerate diagonal matrix.The implications of these models on neutrinoless double beta decay are also discussed.

Let us consider a Majorana mass matrix in $(e,\, \mu,\, \tau)$ basis

$$M_\nu = m_0 \begin{pmatrix} a_{ee} & a_{e\mu} & a_{e\tau} \\ a_{e\mu} & a_{\mu\mu} & a_{\mu\tau} \\ a_{e\tau} & a_{\mu\tau} & a_{\tau\tau} \end{pmatrix}. \tag{4}$$

It is convenient to define the neutrino mixing angles as follows

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \tag{5}$$

where

$$\begin{pmatrix} c_2 c_3 & c_2 s_3 & s_2 e^{i\delta} \\ -c_1 s_3 - s_1 s_2 c_3 e^{i\delta} & c_1 c_3 - s_1 s_2 s_3 e^{i\delta} & s_1 c_2 \\ s_1 s_3 - c_1 s_2 c_3 e^{i\delta} & -s_1 c_3 - c_1 s_2 s_3 e^{i\delta} & c_1 c_2 \end{pmatrix} \tag{6}$$

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with $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$. We shall put δ as well as Majorana phases to be zero. In view of eqs. (1), (2) and (3), we shall take

$$s_2 = 0\,, \qquad c_2 = 1\,.$$

The diagonalization of mass matrix (4) give the relations

$$\begin{aligned} m_1 c_3^2 + m_2 s_3^2 &= a_{ee} \\ m_1 c_1^2 s_3^2 + m_2 c_1^2 c_3^2 + m_3 s_1^2 &= a_{\mu\mu} \\ m_1 s_1^2 s_3^2 + m_2 s_1^2 c_3^2 + m_3 c_1^2 &= a_{\tau\tau} \\ c_1 s_3 c_3 (-m_1 + m_2) &= a_{e\mu} \\ s_1 c_3 s_3 (m_1 - m_2) &= a_{e\tau} \\ -c_1 s_1 \left(m_1 s_3^2 + m_2 c_3^2 - m_3\right) &= a_{\mu\tau}\,. \end{aligned} \tag{7}$$

Taking now $c_1 = 1/\sqrt{2}$, $s_1 = \mp 1/\sqrt{2}$, we obtain

$$\begin{aligned} a_{e\mu} &= \pm a_{e\tau} = \frac{1}{\sqrt{2}} s_3 c_3 \left(-m_1 + m_2\right) \\ a_{\mu\tau} &= \pm \frac{1}{2}\left[\left(m_1 s_3^2 + m_2 c_3^2\right) - m_3\right] \\ a_{\mu\mu} &= a_{\tau\tau} = \frac{1}{2}\left(m_1 s_3^2 + m_2 c_3^2 + m_3\right) \\ a_{ee} &= m_1 c_3^2 + m_2 s_3^2\,. \end{aligned} \tag{8}$$

Further defining

$$\Delta m_{12}^2 = m_2^2 - m_1^2, \Delta m_{23}^2 = m_3^2 - m_2^2, \Delta m_{13}^2 = m_3^2 - m_1^2$$

so that

$$\Delta m_{12}^2 + \Delta m_{23}^2 = \Delta m_{13}^2$$

and in view of $\Delta m_{12}^2 \ll \Delta m_{23}^2$ as indicated in eqs. (1) and (2), we can take

$$m_1 \simeq \pm m_2\,.$$

Thus we have two possibilities for mass matrix $m_1 = m_2 = m_0$; $m_1 = -m_2 = m_0$:

$$\begin{aligned} M_\nu &= m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2}(1+a) & \pm\frac{1}{2}(1-a) \\ 0 & \pm\frac{1}{2}(1-a) & \frac{1}{2}(1+a) \end{pmatrix} \\ M_\nu &= m_0 \begin{pmatrix} \cos 2\theta_3 & -\frac{1}{\sqrt{2}}\sin 2\theta_3 & \mp\frac{1}{\sqrt{2}}\sin 2\theta_3 \\ -\frac{1}{\sqrt{2}}\sin 2\theta_3 & \frac{1}{2}(\cos 2\theta_3 + a) & \pm\frac{1}{2}(\cos 2\theta_3 - a) \\ \mp\frac{1}{\sqrt{2}}\sin 2\theta_3 & \pm\frac{1}{2}(\cos 2\theta_3 - a) & \frac{1}{2}(\cos 2\theta_3 + a) \end{pmatrix}, \end{aligned}$$

where $a = m_3/m_0$. If we do not want to commit to any particular value of θ_3, then we have the first case with the following subcases corresponding to $m_0 = 0$, $a = 0$, $a = -1$, 1, -2, 2 respectively i)

$$\frac{m_3}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad [8]$$

ii)

$$m_0\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad [8]$$

iii)

$$m_0\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

iv)

$$\frac{m_0}{2}\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 3 \\ 0 & 3 & -1 \end{pmatrix} \quad [9]$$

v)

$$\frac{m_0}{2}\begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{pmatrix}.$$

In order to generate Δm^2_{12} and Δm^2_{23}, we will now concentrate on choice (iii) which preserves flavor and add to it a small perturbation which violates flavor in off-diagonal matrix elements:

$$M_\nu = m_0\begin{pmatrix} 1 & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{12} & 1 & \varepsilon_{23} \\ \varepsilon_{13} & \varepsilon_{23} & 1 \end{pmatrix}, \tag{9}$$

where $\varepsilon_{ij} \ll 1$. The diagonalization gives

$$m_i = m_0\,(1 - x_i) \tag{10}$$

where x_i $(i = 1,\, 2,\, 3)$ are roots of cubic equation

$$x^3 - \left(\varepsilon_{12}^2 + \varepsilon_{13}^2 + \varepsilon_{33}^2\right) x + 2\varepsilon_{12}\varepsilon_{13}\varepsilon_{23} = 0\,. \tag{11}$$

The choice $\varepsilon_{12} = \varepsilon_{13} = \varepsilon_{23} = \varepsilon$ will give the roots $(\varepsilon,\, \varepsilon,\, -2\varepsilon)$ and thus will not lift the degeneracy between m_1 and m_2. To lift this degeneracy we take $\varepsilon_{12} = \varepsilon_{13} = \varepsilon + \delta$, $\varepsilon_{23} = \varepsilon$ with $\delta/\varepsilon \ll 1$. Then the roots to the first order in δ/ε are ε, $\varepsilon(1 + \frac{4}{3}\delta/\varepsilon)$, -$2\varepsilon(1 + \frac{2}{3}\delta/\varepsilon)$ so that

$$\begin{aligned} m_1 &= m_0\left[1 - \varepsilon - \frac{4}{3}\delta\right] \\ m_2 &= m_0\,[1 - \varepsilon] \\ m_3 &= m_0\left[1 + 2\varepsilon + \frac{4}{3}\delta\right] \end{aligned} \tag{12}$$

$$\Delta m_{12}^2 \approx \frac{8}{3} m_0^2 \delta (1-\varepsilon) \simeq \frac{8}{3} m_0^2 \delta$$
$$\Delta m_{32}^2 \approx 6 \varepsilon m_0^2 \,. \tag{13}$$

This gives

$$\frac{\delta}{\varepsilon} = \frac{9}{4} \frac{\Delta m_{12}^2}{\Delta m_{32}^2} \simeq 5.9 \times 10^{-2} \tag{14}$$

$$\sqrt{\varepsilon} m_0 \simeq 2.1 \times 10^{-2} \, \mathrm{eV} \tag{15}$$

so that m_0 is not constrained. However, m_0 is constrained by WMAP data [10],$3m_0 < 0.71\,\mathrm{eV}$. When analyzed in conjunction with neutrino oscillation, it is found that mass eigenvalues are essentially degenerate with $3m_0 > 0.4\,\mathrm{eV}$ [11] The above limits put limits on ε : $7.9 \times 10^{-3} < \varepsilon < 2.5 \times 10^{-2}$.

It is pertinent to remark that in view of small mass differences involved [cf. eqs. (1) and (2)], and that analysis of WMAP data in conjunction with neutrino oscillation imply essentially degenerate mass eigenvalues, it is quite natural to remove the degeneracy by small perturbations. Such a procedure should be stable as the parameter ε turns out to be small in view of limit on it given above. The $\delta/\varepsilon \simeq 6 \times 10^{-2}$ may indicate fine tuning but this a problem common to Yukawa couplings even in charged sector, the solution of which has so far evaded us.

Next we consider an extension of the standard electroweak model to

$$G \equiv \mathrm{SU}_L(2) \times \mathrm{U}_e(1) \times \mathrm{U}_\mu(1) \times \mathrm{U}_\tau(1)$$

which naturally gives an off diagonal mass matrix for light neutrinos [12]. In addition to usual fermions $L_i = \begin{pmatrix} \nu^i \\ e^i \end{pmatrix}_L$, e_R^i, there are three right handed $\mathrm{SU}_L(2)$ -singlet neutrinos N_R^i which carry $\mathrm{U}_i(1)$ quantum numbers $(-1, 1, 0)$, $(1, 0, -1)$ and $(0, -1, 1)$. Further in addition to $\mathrm{SU}_L(2)$ Higgs doublets, there are three Higgs $\mathrm{SU}_L(2)$ singlets Σ^i with $\mathrm{U}_i(1)$ quantum numbers $(1, -1, 0)$, $(1, 0, -1)$ and $(0, 1, -1)$. In order to introduce diagonal terms we also introduce a right handed neutrino N and a corresponding Higgs boson Σ which are $\mathrm{SU}_L(2)$ and $\mathrm{U}_i(1)$ singlets. The symmetry is spontaneously broken by giving vacuum expectation values to Higgs boson $\phi^{(i)}$, $\Sigma^{(i)}$ and Σ :

$$\left\langle \phi^{(i)} \right\rangle = \frac{v_i}{\sqrt{2}}, \left\langle \Sigma^{(i)} \right\rangle = \frac{\Lambda_i}{\sqrt{2}}, \langle \Sigma \rangle = \frac{\Lambda}{\sqrt{2}}\,, \tag{16}$$

where for simplicity we shall take $v_1 = v_2 = v_3 = v$ and $\Lambda_1 = \Lambda_2 = \Lambda_3 = \Lambda$ (any difference can be absorbed in the corresponding Yukawa couplings of fermions with Higgs bosons). The resulting mass matrix through seesaw mechanism for light neutrinos is

$$M_\nu = \frac{v^2}{2M_R} \begin{pmatrix} h_1^{(1)} h_1^{(1)} & h_1^{(2)} h_2^{(3)} & h_1^{(2)} h_3^{(1)} \\ h_1^{(2)} h_2^{(3)} & h_2^{(2)} h_2^{(2)} & h_2^{(3)} h_3^{(1)} \\ h_1^{(2)} h_3^{(1)} & h_2^{(3)} h_3^{(1)} & h_3^{(3)} h_3^{(3)} \end{pmatrix} \tag{17}$$

where $M_R = \frac{f\Lambda}{\sqrt{2}}$. We now put $h_i^{(i)} = h$, $\frac{h_1^{(2)} h_2^{(3)}}{h^2} = \varepsilon_{12}$, $\frac{h_1^{(2)} h_3^{(1)}}{h^2} = \varepsilon_{13}$, $\frac{h_2^{(3)} h_3^{(1)}}{h^2} = \varepsilon_{23}$. This gives the desired matrix (9) with $m_0 = \frac{h^2 v^2}{2M_R}$. If h is of the order of 1 and m_0 is few electron volts, then one requires $M_R \simeq 10^{13}$ GeV.

Another simple way to obtain the neutrino mass matrix (9) is to include a Higgs triplet $\Delta(3,2) : (\Delta^{++},\ \Delta^{+},\ \Delta^{0})$ with the interaction[13, 14]

$$\mathcal{L} = h_{ij} L_i^T i\tau_2 \left(\vec{\tau}.\vec{\Delta}\right) L_j \tag{18}$$

generating neutrino mass matrix

$$\left(M_\nu\right)_{ij} = 2h_{ij} \left\langle \Delta^0 \right\rangle . \tag{19}$$

Taking $h_{11} = h_{22} = h_{33} = h$ and $h_{12}/h = \varepsilon_{12}$, $h_{13}/h = \varepsilon_{13}$, and $h_{23}/h = \varepsilon_{23}$; $m_0 = 2h\langle\Delta^0\rangle$ we get the desired mass matrix. If h is of order unity, and m_0 is a few electron volts, then $\langle\Delta^0\rangle$ is of order of eV, which appears to be unnatural. It is well known that the coupling of Δ to gauge field will alter the ρ parameter ($\rho \equiv \frac{m_w^2}{m_z^2 \cos^2\theta_w}$) by

$$\Delta\rho \equiv (1-\rho) = 4\frac{\left\langle\Delta^0\right\rangle^2}{v^2} . \tag{20}$$

The present limit on $\Delta\rho$ (≤ 0.02) only requires $\langle\Delta^0\rangle < 25\,\mathrm{GeV}$. To give a more natural look to small value of Δ^0, lepton flavor violating term of the form $\mu(\phi^\dagger\Delta\tilde{\phi}) + H.C$ is added to L conserving Higgs potential. Thus [14]

$$\left\langle\Delta^0\right\rangle = \frac{\mu v^2}{2M_\Delta^2} \simeq 3\times 10^4 \mu \left(\frac{\mathrm{GeV}^2}{M_\Delta^2}\right) . \tag{21}$$

For $\langle\Delta^0\rangle \sim$ eVμ has to be quite small ($\simeq 3\,\mathrm{eV}$) if M_Δ is $\sim 1\,\mathrm{TeV}$. If $\mu \sim 1\,\mathrm{GeV}$, $M_\Delta \simeq 3\times 10^6\,\mathrm{GeV}$.

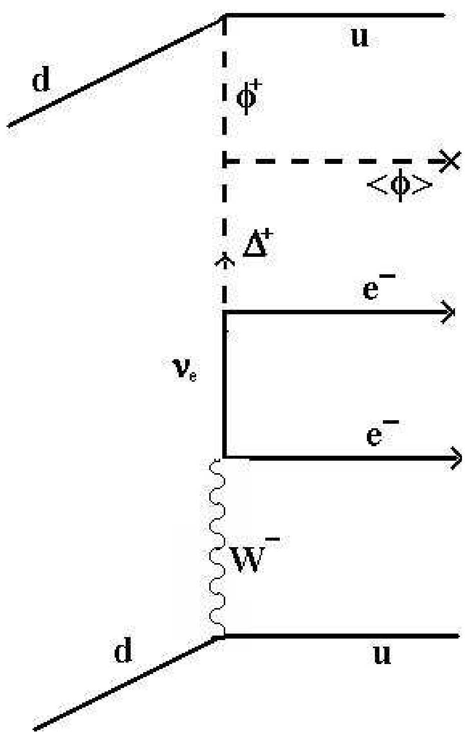

Figure 1: The vector-scalar exchange diagram for $\beta\beta_{0\nu}$.

Finally we discuss the implications of above models in neutrinoless double beta decay. In the first model at tree level there is only the standard contribution through light Majorana ν_e. In the second model, there could be additional contribution at tree level through diagram in figure 1.

This give rise to an effect $\Delta L = 2$ interaction which can be written as [15]

$$H_{\Delta L=2} = -G_F^2 \epsilon_1^{ee} \bar{u}\left(1+\gamma_5\right) d\bar{u}\gamma_\mu \left(1-\gamma_5\right) d\bar{e}\gamma^\mu \left(1-\gamma_5\right) \frac{1}{\gamma - q} C\bar{e}^T , \tag{22}$$

where $[h_{11} = h]$

$$\epsilon_1^{ee} = \frac{h_u\left(h\mu v/\sqrt{2}\right)}{u\sqrt{2}G_F m_\Delta^2 m_H^2} . \tag{23}$$

On using eq. (21) and that $m_0 = 2h(\Delta^0)$, we obtain

$$\epsilon_1^{ee} = \frac{\left(h_u\right) m_0}{8\sqrt{2}G_F m_H^2 v/\sqrt{2}} .$$

With $\frac{v}{\sqrt{2}} \simeq 175\,\mathrm{GeV}$, we obtain $\epsilon_1^{ee} = h_u(m_0/\,\mathrm{eV})(5\times 10^{-10})$. One may expect [15] $h_u = m_u/m_W \simeq 5{\times}10^{-5}$ so that ϵ_1^{ee} is much too small compared to the upper limit on $\epsilon_1^{ee}(\leq 10^{-8})$ found in [15] from the neutrino mass contribution to $\beta\beta_0\nu$ ($m_\nu < 1\,\mathrm{eV}$).

To sum up we have shown that it is possible to interpret the neutrino mass differences and mixings with approximate flavor symmetry. The small flavor violation, manifested in neutrino mass differences, is in the Yukawa coupling constants h_{ij} which determine the off-diagonal matrix elements with $\varepsilon_{ij}(i \neq j) \equiv h_{ij}^2(i \neq j)/h^2 \sim 10^{-3}$. This may have some phenomenological implications. We have also discussed two modest extensions of the standard model which can embed the resulting neutrino mass matrix.

Note added: it has been called to my attention by A. Zee that the matrix (9) in this paper is similar to $\widetilde{M}_0$ with $y = -1$, given by X.G. He and A. Zee [16]. Our motivation is rather different.

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References

[1] PARTICLE DATA GROUP collaboration, K. Hagiwara et al., *Review of particle physics*, *Phys. Rev.* **D 66** (2002) 010001.

[2] SNO collaboration, Q.R. Ahmad et al., *Direct evidence for neutrino flavor transformation from neutral-current interactions in the Sudbury Neutrino Observatory*, *Phys. Rev. Lett.* **89** (2002) 011301 [nucl-ex/0204008]; *Measurement of day and night neutrino energy spectra at SNO and constraints on neutrino mixing parameters*, *Phys. Rev. Lett.* **89** (2002) 011302 [nucl-ex/0204009];
SUPER-KAMIOKANDE collaboration, S. Fukuda et al., *Determination of solar neutrino oscillation parameters using 1496 days of Super-Kamiokande-I data*, *Phys. Lett.* **B 539** (2002) 179 [hep-ex/0205075];
B.T. Cleveland et al., *Measurement of the solar electron neutrino flux with the homestake chlorine detector*, *Astrophys. J.* **496** (1998) 505;
R. Davis, *A review of the Homestake solar neutrino experiment*, *Prog. Part. Nucl. Phys.* **32** (1994) 13;
SAGE collaboration, D.N. Abdurashitov et al., *Measurement of the solar neutrino capture rate with gallium metal*, *Phys. Rev.* **C 60** (1999) 055801 [astro-ph/9907113];
KASCADE collaboration, K.H. Kampert et al., *The physics of the knee in the cosmic ray spectrum*, astro-ph/0204205;
GALLEX collaboration, W. Hampel et al., *Gallex solar neutrino observations: results for Gallex IV*, *Phys. Lett.* **B 447** (1999) 127;
GNO collaboration, C. Cattadori, *Update of solar neutrino interaction rate measurements from GNO at LNGS*, in *Topics in astroparticle and underground physics*, Assergi, Italy, 8-12 Sep 2001, *Nucl. Phys.* **110** *(Proc. Suppl.)* (2002) 311.

[3] KAMIOKANDE collaboration, Y. Fukuda et al., *Atmospheric muon-neutrino/electron-neutrino ratio in the multigev energy range*, *Phys. Lett.* **B 335** (1994) 237;
IBM collaboration, R. Becker-Szendy et al., *Nucl. Phys.* **38** *(Proc. Suppl.)* (1995) 331;

SOUDAN-2 collaboration, W.W.M. Allison et al., *The atmospheric neutrino flavor ratio from a 3.9 fiducial kiloton-year exposure of Soudan 2*, *Phys. Lett.* **B 449** (1999) 137 [hep-ex/9901024];
MACRO collaboration, M. Ambrosio et al., *Measurement of the atmospheric neutrino-induced upgoing muon flux using macro*, *Phys. Lett.* **B 434** (1998) 451 [hep-ex/9807005];
MACRO collaboration, M. Sioli, *Atmospheric neutrino oscillations with macro*, hep-ex/0211030.

[4] KAMLAND collaboration, K. Eguchi et al., *First results from KamLAND: evidence for reactor anti-neutrino disappearance*, *Phys. Rev. Lett.* **90** (2003) 021802 [hep-ex/0212021].

[5] K2K collaboration, M.H. Ahn et al., *Indications of neutrino oscillation in a 250 Km long baseline experiment*, *Phys. Rev. Lett.* **90** (2003) 041801 [hep-ex/0212007].

[6] See e.g. in the proceedings of the X international worshop on neutrino telescopes, Venezia, march 11-14, 2003. Transparencies are available at the internet address http://axpd24.pd.infn.it/conference2003/venice03.html.

[7] CHOOZ collaboration, M. Apollonio et. al., *Limits on neutrino oscillations from the CHOOZ experiment*, *Phys. Lett.* **B 466** (1999) 415 [hep-ex/9907037].

[8] S.F. King, *Phenomenological and cosmological implications of neutrino oscillations*, *J. Phys.* **G 29** (2003) 1551 [hep-ph/0210081].

[9] X.-G. He and A. Zee, *Neutrino masses with 'zero sum' condition: $m_{\nu_1} + m_{\nu_2} + m_{\nu_3} = 0$*, *Phys. Rev.* **D 68** (2003) 037302 [hep-ph/0302201].

[10] D.N. Spergel et al., *First year Wilkinson microwave anisotropy probe (WMAP) observations: determination of cosmological parameters*, *Astrophys. J. Suppl.* **148** (2003) 175 [astro-ph/0302209].

[11] O. Elgaroy and O. Lahav, *The role of priors in deriving upper limits on neutrino masses from the 2DFGRS and WMAP*, *JCAP* **0304** (2003) 004 [astro-ph/0303089].

[12] Fayyazuddin and Riazuddin, *Neutrino masses and electron - muon - tau - lepton universality*, *Phys. Rev.* **D 35** (1987) 2201;
Riazuddin, *Off-diagonal structure of neutrino mass matrix in see-saw mechanism and electron muon tau lepton universality*, *Phys. Rev.* **D 63** (2001) 033003 [hep-ph/0007146].

[13] J. Schechter and J.W.F. Valle, *Neutrino masses in* $\mathrm{SU}(2) \times \mathrm{U}(1)$ *theories*, *Phys. Rev.* **D 22** (1980) 2227;
Riazuddin, R.E. Marshak and R.N. Mohapatra, *Majorana neutrinos and low-energy tests of electroweak models*, *Phys. Rev.* **D 24** (1981) 1310;
E. Ma, M. Raidal and U. Sarkar, *Verifiable model of neutrino masses from large extra dimensions*, *Phys. Rev. Lett.* **85** (2000) 3769 [hep-ph/0006046].

[14] E. Ma, *Neutrino mass from triplet and doublet scalars at the TeV scale*, *Phys. Rev.* **D 66** (2002) 037301 [hep-ph/0204013].

[15] K.S. Babu and R.N. Mohapatra, *New vector-scalar contributions to neutrinoless double beta decay and constraints on R-parity violation*, *Phys. Rev. Lett.* **75** (1995) 2276 [hep-ph/9506354].

[16] X.G. He and A. Zee, *Some simple mixing and mass matrices for neutrinos*, *Phys. Lett.* **B 560** (2003) 87 [hep-ph/0301092];
A. Zee, *Some remarks on neutrino oscillation and masses*, hep-ph/0307155.

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